Micro and Nano-Tomography
Use of X-rays for imaging

Wilhelm Conrad Roentgen
first Nobel prize in physics (1901)
Compressing 3-d information into a 2-d image
Development of tomography

Sir Godfrey N. Hounsfield

Allan M. Cormack

Nobel Prize in Physiology or Medicine 1979
Tomography comes from the Greek word tomos (slice) and it has the goal of imaging an object by taking measurements from “slices” of its cross-section.
Configuration

X-ray source

Detector

y

x

y

x

Detector
Radiographic (left) and CT image (right) images of the loaded fiber-reinforced concrete cylinder

Finding many bars

Radiographic (left) and CT (right) image.

Large samples

reinforced concrete column

radiography

3-D tomography

images courtesy of Dr. Harry Martz from LLNL
• Synchroton produces x-rays that are parallel to each other.

• X-rays that penetrate through the object – i.e. are not absorbed - are converted to visible light in a scintillator crystal.

• CCD sensor records the visible light image.
Sinogram from x-ray transmission image

Each point in the object draws a sine curve in the sinogram

Transmission image at fixed $\theta$

Sinogram: at fixed $y$, rotate object
A sinogram is a collection of projections measured from all angles $\theta$:

$$p(s, \theta) = \int \int f(x,y) \delta[s-x\cos(\theta)-y\sin(\theta)] \, dx \, dy$$

where $\delta(x)=0$ when $x \neq 0$, and $\delta(x)=1$ when $x=0$

Note: $p(-s, \theta) = p(s, \theta+\pi)$
Projection theorem
(also known as Fourier slice theorem)

The 1-dim Fourier transform of the projection \( p(s,\theta) \) with respect to \( s \) is identical to the 2-dim Fourier transform of the object \( f(x,y) \) evaluated in polar coordinates at angle \( \theta \):

\[
P(\omega_s,\theta) = F(\omega_s \cos \theta, \omega_s \sin \theta)
\]

where \( P(\omega_s,\theta) = F_s \{ p(s,\theta) \} \) and

\[
F(\omega_s \cos \theta, \omega_s \sin \theta) = F_{x,y} \{ f(x,y) \} \mid \omega_x = \omega_s \cos \theta, \omega_y = \omega_s \sin \theta
\]
What does this mean?

• Take a 1-D Fourier transform of each line in a sinogram
• Arrange the 1-D Fourier transforms into polar coordinates
• Interpolate from polar to Cartesian coordinates
• Take 2-D inverse Fourier transform \( \Rightarrow f(x,y) \)
Proof of projection theorem:

Fourier transform $\mathcal{P}(\omega_s, \theta)$ of projection $p(s, \theta)$:

$$\mathcal{P}(\omega_s, \theta) = \mathcal{F}_s \{p(s, \theta)\} = \int e^{-i2\pi\omega_s s} p(s, \theta) \, ds =$$

$$\int e^{-i2\pi\omega_s s} \left\{ \int \int f(x,y) \delta[s-x \cos(\theta)-y \sin(\theta)] \, dx \, dy \right\} \, ds =$$

$$\int \int f(x,y) \left\{ \int e^{-i2\pi\omega_s s} \delta[s-x \cos(\theta)-y \sin(\theta)] \, ds \right\} \, dx \, dy$$

$$\int \int f(x,y) \, e^{-i2\pi\omega_s [x \cos(\theta)+y \sin(\theta)]} \, dx \, dy =$$

$$\mathcal{F}_{x,y} \{f(x,y)\} \bigg|_{\omega_x=\omega_s \cos \theta, \ \omega_y=\omega_s \sin \theta} = \mathcal{F}(\omega_s \cos \theta, \ \omega_s \sin \theta)$$
Image reconstruction algorithms

Previous slide’s Direct Fourier Reconstruction suffers from:

i) uneven data density – more data points near the origin, less near edges

ii) interpolation inaccuracies in polar-to-cartesian conversion

Improved reconstruction algorithm needed

=> Backprojection of Filtered Projections
Backprojection of Filtered Projections

\[ f(x, y) = \int_0^\infty \int_0^\infty e^{i2\pi(\omega_x x + \omega_y y)} F(\omega_x, \omega_y) \, d\omega_x \, d\omega_y \]

Inverse Fourier Transf.

First we do cartesian-to-polar coordinate transform:

\[ \omega_x = \omega_s \cos \theta, \quad \omega_y = \omega_s \sin \theta \]

\[ d\omega_x \, d\omega_y = \omega_s \, d\omega_s \, d\theta \]

and we get:

\[ f(x, y) = \int_0^\infty \int_0^\infty e^{i2\pi \omega_s (x \cos \theta + y \sin \theta)} F(\omega_s \cos \theta, \omega_s \sin \theta) \omega_s \, d\omega_s \, d\theta \]

\[ = \int_0^\infty \int_{-\infty}^{\infty} e^{i2\pi \omega_s (x \cos \theta + y \sin \theta)} P(\omega_s, \theta) |\omega_s| \, d\omega_s \, d\theta \]
Here we used the projection theorem:

$$P(\omega_s, \theta) = F(\omega_s \sin \theta, \omega_s \cos \theta)$$

And the symmetry property of projections:

$$p(-s, \theta) = p(s, \theta + \pi) \implies P(-\omega_s, \theta) = P(\omega_s, \theta + \pi)$$

But because the inner integral is the inverse Fourier transform of the product of $P(\omega_s, \theta)$ and $|\omega_s|$ we can identify **ramp-filtering**:

$$p(\theta) = \int_{-\infty}^{\infty} e^{i2\pi \omega_s s} P(\omega_s, \theta) |\omega_s| d\omega_s$$

where $p(\theta)$ is the filtered projection, and $|\omega_s|$ is the ramp filter
Tomographic Image Reconstruction with Backprojection of Filtered Projections

1. Filter projections \( p(s, \theta) \) with ramp filter:
   - Fourier transform with respect to \( s \)
   - Multiply with \( |\omega_s| \)
   - Inverse Fourier transform

2. Backproject filtered projections \( p(s, \theta) \):
   - Evaluate at \( s = x \cos \theta + y \sin \theta \)
   - Integrate over \( \theta \) from 0 to \( \pi \)

\[
f(x, y) = \int_{0}^{\pi} p(s, \theta) \big|_{s=x \cos \theta + y \sin \theta} d\theta
\]
Microtomography

2-phase segmentation of the carbon steel (CS) and the ferritic stainless steel (FSS) samples at different times. The steel is in light grey, the migrating corrosion products are in orange, and the propagating cracks and the air voids are in dark grey.

3D images showing fiber distribution in a cement mortar:

(a) hybrid fiber block

(b) PP fiber block.

Daniel Hernandez Cruz, Craig Hargis; Sungchul Bae, Pierre A Itty, Cagla Meral, Jolee Dominowsky, Michael J Radler, David A Kilcoyne, Paulo J.M Monteiro, Multiscale characterization of chemical-mechanical interactions between polymer fibers and cementitious matrix, Cement and Concrete Composites, Volume 48, April 2014, Pages 9–18.
3D-reconstruction of chert in it_06: a subvolume rendering showing dissolution zone in chert (blue) and debonding (red); b iso-surfaces rendering of same volume; c rendering of the subvolume with microcracks (red) emanating from the aggregate particle; d iso-surfaces rendering of same volume

a) Volume rendering of chert showing signs of silica dissolution; b) iso-surfaces rendering of chert showing silica dissolution/alteration (blue) and porosity (red)

Challenges of Nanotomography

- Alignment of the images
- Stability of the system
- Limited angle tomography when using flat sample holders
Nanotomography of $C_3S$ Hydration

S. Brisard, R. S. Chae, I. Bihannic, L. Michot, P. Guttmann, J. Thieme, G. Schneider, P.J.M. Monteiro and P. Levitz, Morphological quantification of hierarchical geomaterials by x-ray nano-CT bridges the gap from nano to micro length-scales, American Mineralogist, 97 2-3 480-483, FEB-MAR 2012.
Nanotomography of the Roman concrete