

# Scaling Laws in Nanomechanics

I particularly liked this line of research because it allowed me to work with G.I. Barenblatt who, besides being the brightest mathematician that I ever met, is a fascinating person. Our meetings were intense and I become a much better researcher thanks to him.

# Our joint publication:

- G. I. Barenblatt and P. J. M. Monteiro, Scaling Laws in Nanomechanics, Physical Mesomechanics, 13, 245-248 (2010).
- P.J.M. Monteiro, C. H. Rycroft , and G. I. Barenblatt, Fluid and gas flow in nano-porous media: A mathematical model and applications, PNAS, vol 109. No.50., 20309–20313, 2012.
- G.I. Barenblatt, P.J.M. Monteiro, C. H. Rycroft, On a boundary layer problem related to the gas flow in shales, J Eng Math (2014) 84:11–18.

# Introduction

- The energy of radiation is transferred  $h\nu$  not continuously, but by portions (i.e., quantum) equal to  $h\nu$ , where  $\nu$  is the frequency of the radiation and  $h$  is a fundamental constant.
- Quantum phenomena should enter the mechanisms of the deformation and fracture at a nanoscale.

# Parameters at nanoscale

- Planck constant should enter the governing parameters at these scales.
- The other quantities influencing the deformation and fracture at these scales are: Young's modulus  $E$  and density, because the formation of defects at a small scale is always a dynamical process even if the global, macroscopic process of deformation is static.

# Parameters at nanoscale

- Planck's constant is equal to  $6.626 \times 10^{-34} \text{ m}^2\text{kg/s}$ .
- The density for metals, concrete, etc., in ordinary conditions (materials that are not highly compressed, e.g., by converging shock waves) varies; for platinum equals  $21.0 \times 10^3 \text{ kg/m}^3$  for tungsten and for  $1.0 \times 10^3 \text{ kg/m}^3$  water.

# Consider the quantity

$$\frac{h}{(\rho E)^{1/2}} = \frac{h}{\rho (E/\rho)^{1/2}}$$

Which is measured in  $\text{m}^4$

So the quantity

$$\lambda = \left( \frac{h}{\rho (E/\rho)^{1/2}} \right)^{1/4}$$

Is measured in meter.

## An important observation:

- Remarkably, this quantity is practically the same for a wide spectrum of materials from platinum to concrete and water (note that it is critical to observe the small exponent  $\frac{1}{4}$ ).



# For most materials

$$\lambda = \left( \frac{h}{\rho (E/\rho)^{1/2}} \right)^{1/4} = A (10^{-40})^{1/4} \cong 10^{-10} \text{ m}$$

# Physical meaning

- It is the length of the “pixel” accepted in nanomechanics.
- Note that the length is a fundamental (not arbitrary assigned as one micron for micromechanics) physical quantity.
- In particular, the normal interatomic distances in crystal lattice have this order of magnitude.

## Consequence:

- the parameter  $\lambda$  should enter the governing parameters that determine the properties of deformation and strength of structures.

## Consequence:

- Therefore the parameter  $\Lambda/\lambda$  where  $\Lambda$  is the length scale of a specimen or of a structure should be included in the laws of scaling.

## Consequence:

- Generally speaking, this means that the scaling effect could be observed in testing the macroscopic, large specimens or structures.
- It seems to be a plausible way to incorporate the nano-mechanical (quantum mechanical) effects on fatigue, fracture, plasticity, creep and other mechanical properties usually attributed to macroscopically large specimens and structures.