DEVELOPMENT AND ANALYSIS OF A DEVICE FOR MEASURING COMPRESSIVE STRESS IN CONCRETE

by

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May 10, 1939

Professor G. W. Swett
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge, Massachusetts

Dear Sir:

In partial fulfillment of the requirements for the degree of Doctor of Science, I submit herewith my thesis entitled "Development and Analysis of a Device for Measuring Compressive Stress in Concrete."

Respectfully submitted,

Roy W. Carlson

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Abstract

A 12-year development of a device called a "stress meter" is described. This device is shown to be capable of indicating compressive stress in concrete substantially independent of differences in deformation between the device and the concrete in which it is embedded.

The stress meter is a fluid-filled diaphragm, so designed that the pressure in the fluid is always substantially equal to the pressure of the surrounding concrete lying against the faces of the diaphragm. The fluid is in contact with a small internal plate, which is deflected elastically in direct proportion to the intensity of stress or pressure. A small electric strain meter, mounted on one face of the diaphragm, detects the deflection of the internal plate and by means of conductor cable and testing set indicates the deflection and hence the stress at any convenient termination of the cable.

Analyses are described which substantiate the claim that the stress can be indicated with only a small error when the diaphragm tends to remain constant in thickness while the surrounding concrete contracts. The effects of changes in temperature and in modulus of elasticity also are discussed.

Finally, results are presented of tests on stress meters embedded in large concrete cylinders subjected to independently-measured stress and strain. These results support the results of the analyses.

I. INTRODUCTION

A. The Need for a Stress-Measuring Device

The design of concrete structures and the evaluation of their margins of safety are based on calculated stresses, not on deformations. Measurements on structures aimed toward determining their safety and the manner in which they support the applied loads have usually been confined to deformations, since stress measurement has heretofore been impossible. From measured deformations, attempts have been made to calculate the stresses in devious ways.

The relation between stress and deformation in a material like concrete is usually complicated by the simultaneous occurrence of deformations arising from causes other than load. Consequently, the relation is not a simple one. Furthermore, the relation between stress and deformation involves the age of the concrete, the duration of the stress, and the magnitude of the stress. Before proceeding with the explanation of a device for measuring direct stress, therefore, the limitations of measurements of deformation as a means of determining stress will be further outlined. Discussion of these may clarify the need for a separate device for measuring stress.

Firstly, the modulus of elasticity of concrete gradually increases as the concrete hardens and only after a month or more is the modulus substantially constant. That is to say, the deformation due to the quick application of a given stress becomes less and less as the concrete becomes older. Thus, even for the simple task of computing the magnitude of stress which is applied quickly, the observed strain must be multiplied by a modulus of elasticity which depends upon the age of the concrete. If this were the only factor to complicate the problem of translating deformation into stress, it would not be too difficult to apply to each observed change in deformation the appropriate modulus of elasticity, as measured at various ages on specimens of corresponding concrete. But there are other complications.

Secondly, there is the abnormal effect of temperature changes upon deformations in concrete. In general, concrete expands with each degree of rise in temperature more than it contracts upon cooling, with the result that alternations in temperature cause a "growth" of the dimensions. Furthermore, in the early stages of hardening, there is an additional growth due to hydration of the hard-burned oxides of magnesia and lime that are contained in ordinary cement, and this growth may make the apparent coefficient of expansion as much as 50 to 100 per cent greater for the first rise in temperature than for later heating or cooling. Only if concrete is made from cement that is free from the hard-burned oxides of magnesia and lime is this growth effect absent.

The increased difficulty of translating deformation into stress that is added by these "growth" effects is still not too great. Stress-free specimens of identical concrete can be provided in cavities within the body of a concrete structure and these specimens will respond to all causes other than stress. Deformations of these specimens, as measured by means of embedded electric strain meters, can be assumed to be equal to those of the neighboring concrete of the structure except for the effect of stress. When these deformations are subtracted, therefore, from those observed at neighboring points of the structure itself, the remainder can be assumed to result only from stress. The stress could then be computed by multiplying this remainder by the appropriate modulus of elasticity, provided no other factors were operative.

The problem is not yet solved, because there is the plasticity of the concrete to consider. Concrete continues to deform under stress at a rate that depends upon the age, strength, and intensity of stress. Although the plasticity gradually diminishes with increasing age, even after a month of hardening, the plastic deformation that accumulates when the stress is continually applied for about two months will usually be as great as the instantaneous, elastic deformation. After learning that plastic deformation is approximately proportional to the intensity of stress, one can resort to the use of a lower, spurious modulus of elasticity, called the "sustained modulus of elasticity," and thus translate deformation into stress with automatic allowance for plastic flow. The simplicity of this attack is upset by the necessity of knowing the sustained modulus of elasticity not only for the particular concrete, but also for the particular age, time interval of loading, and temperature of hydration. Furthermore, the simple expedient of using a sustained modulus of elasticity does not take into account the plastic flow due to stress already existing; therefore, a separate correction must be made for each time interval. The conversion of deformation into stress must be a step-by-step process, considering one time interval and then another.

Finally, the contraction of concrete upon drying is so great that whenever drying occurs in appreciable amount, the contraction due to drying is likely to outweigh deformation due to all other causes. Fortunately, the diffusion of moisture from concrete is such a slow process that drying is not a major factor except in thin-walled structures.

Any one of the several difficulties in the way of translating deformation of concrete into stress might not be too much for the engineer, but the combination of all of them discourages the use of this means of determining stress. The limitations of so-called "strain" measurements as a means of determining stress were first realized by the writer in 1926 during the testing of the full-scale experimental arch dam at Stevenson Creek, California. Strains were measured both internally and on the surfaces of the dam on almost 1000 gauge lines. Although as computed from changes in deformation, the changes in stress due to the rapid filling of the reservoir were in reasonable agreement with corresponding stresses in an elastic model of the dam, cracking occurred with no apparent relation to the tensile stress of the concrete. In other words, while the rapid change in stress could be learned from the deformation measurements, the actual, existing stresses were not revealed. It was then that the writer realized the need for a device to measure stress substantially independent of deformation, and he began the development of the "stress meter." Thus, the stress meter has been under development for about 12 years. Three of the 12 years of development were spent in the laboratories of the Massachusetts Institute of Technology.

B. <u>Why Measurements of Deformation are Made</u>

Up to the present time, no device has been available for measuring stress in concrete. Therefore, engineers have resorted to the measurement of deformations by the use of electric strain meters, despite the difficulties of translating the results into stress.

Measurements of deformation can be of real value in revealing probable changes in stress over short periods of time and at later ages when the concrete has attained a fair degree of elasticity and when temperature fluctuations are small. Designing engineers usually are more concerned with changes in stress due to load than with actual stresses, and in many cases, the deformations give reliable indications of these load-caused changes in stress.

Concurrently with the development of the "stress meter," the writer has developed the so-called "elastic-wire" electric strain meter, which is now being used almost exclusively in this country for measuring internal deformations of concrete. This strain meter, which functions by the change in resistance of highly elastic wire, seems to be the best device for measuring strain within concrete up to the present time. There was no great difficulty in developing the strain meter as compared with the stress meter, and therefore only a few years of gradual development were required to make it an accepted instrument. More than 2000 strain meters have been used, although they have never been advertised for sale. Among the more important structures in which strain meters have been used, may be mentioned Boulder, Norris, Hiwassee, Tygart, and Grand Coulee Dams, Colorado River Aqueduct, and San Francisco-Oakland Bridge. They have also been used in several foreign countries. One reason for not advertising strain meters for sale has been the belief that they may eventually be supplanted largely by the stress meter. Also, the interpretation of results from strain meters has been so difficult that it has not seemed wise to encourage promiscuous installations.

The following brief description of the strain meter is given because this device, in modified form, is an important part of the present stress meter. An isometric view of the standard strain meter as drawn by Jerome Raphael (now with the United States Bureau of Reclamation) is presented in Fig. 1. From this figure it may be seen that the strain meter contains two coils of wire looped around porcelain spools. The wire is a very fine size of music wire having a tensile strength in the neighborhood of 700,000 pounds per square inch. The mounting of the porcelain spools on the steel frame is such that existing tension in the outer coil is decreased as the ends of the meter are brought closer together, while tension in the inner coil is increased. Because of the linear relationship between tension and electrical resistance, the resistance ratio of the two coils is changed in direct proportion to the change in gauge length. Direct measurement of resistance ratio is made by connecting to a testing set so as to form a Wheatstone-bridge circuit in which two of the four arms are in the testing set and the others are in the strain meter. The resistance ratio is not affected by changes in temperature even though the resistances themselves are affected. Quite conveniently, temperature can be determined by a measurement on the same coils of wire and by means of the same testing set. For this purpose connections are made so as to permit measurement of the combined resistance of both coils, because as a result of the compensating effect of any increase of tension in one coil being offset by a decrease in tension of the other, this resistance is substantially independent of the tensions in the wires. For further details of the strain meter, reference is made to published papers.[1]

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[1] Carlson, R. W., "Five Years Improvement of the Elastic-Wire Strain Meter," Eng. News-Record, May 16, 1935.

Davis, R. E. and Carlson, R. W., "The Electric Strain Meter and Its Use in Measuring Internal Strain," Proc. A.S.T.M., 1932.

II. DEVELOPMENT OF STRESS METER

A. Conception of the Stress Meter

The conception of the stress meter may be appreciated most readily from a roundabout explanation. Consider what would happen if a well-defined block of concrete could be cut and removed from the interior of a concrete mass so as to leave a cavity. For convenience, let us define the imaginary walls of the block before its removal as the walls of the "uncut cavity." It will also be convenient later to depart from the usual definition of the word cavity and to speak not only of an "empty cavity," but also of the cavity filled with an elastic body.

If stress existed in the concrete before the cavity were cut, then when the cut was made, local relief of stress would deflect the walls of the cavity relative to the walls of the uncut cavity. The walls would be returned to the "uncut" locations only by the application of exactly the same stress as existed before the cavity was cut. The reasoning is the same if the cavity be assumed to exist before the stress is applied; if the walls of the cavity were to be subjected to just sufficient stress to maintain them nearly where they would be if the cavity were uncut, the required stress would be nearly the same as that occurring if the cavity were uncut. The converse of this statement is more important; if the walls of the cavity were to be maintained nearly where they would be if the cavity were uncut, the stress on the walls of the cavity would of necessity be nearly that of the undisturbed concrete at that location. There is nothing startling in this view; it is merely an introduction leading up to the otherwise-difficult explanation of how a stress measurement can be almost independent of the magnitude of the attendant strain.

The conditions in the assumed cavity become interesting when the cavity takes the shape of a thin plate or disc. If no stress is applied to the walls of the cavity, the flat faces are deflected a relatively large amount by stress in the surrounding concrete. For example, consider a cavity 10 inches in diameter and one-tenth of an inch thick. If a stress of 100 p.s.i. is applied to the surrounding concrete in a direction perpendicular to the plane of the cavity, the deflection of the midpoint of each face (according to equations presented later and assuming a modulus of elasticity of 2,000,000 p.s.i. for the concrete) will be 0.001 inch. If now the cavity were fitted with a disc of the concrete before the application of stress, the corresponding movement of either face of the disc due to direct strain, would be only 0.000,002.5 inch (.05 inch multiplied by 50 millionths strain). In other words, the faces of the cavity when empty would be deflected by stress in the surrounding concrete 400 times as much as would the faces when the cavity was fitted with a concrete disc. As a further illustration, if the cavity were instead fitted with a disc of material twice as compressible as the concrete, the movement of the faces due to stress would still be small, being about 100 per cent more than that of the concrete disc, but the excess being only one-quarter of one per cent of the movement of the faces of the empty cavity. In other words, a disc of material having a modulus of elasticity only one-half that of the concrete, when fitted to the cavity, maintains its faces nearly where they would be were the cavity uncut. The stress through this more-yielding disc would, therefore, be almost exactly the same as though it had the same rigidity as the concrete. Likewise, if this particular cavity were fitted with a disc possessing infinite rigidity, the faces would remain after application of stress to the surrounding concrete nearly where they would be for the uncut cavity. That is to say, the faces of the filling would not move at all, whereas the faces of the uncut cavity would move 0.000,002.5 inch and the faces of the empty cavity would move 0.001 inch. Thus, the deficiency in movement of the faces of the cavity filled with material of infinite rigidity would be only 1/4 of 1 per cent as much as the excess of movement of the faces of the empty cavity. It follows that the stress through the non-yielding disc would be nearly the same as though the cavity were uncut. From these illustrations, the conclusion is reached that the stress and consequent strain in a thin disc of elastic material will bear a close relation with the stress but not necessarily with the deformation in surrounding concrete.

The stress meter takes advantage of the fact, which will be proved by analyses later, that when a thin plate is cast in concrete, the stress transmitted through the plate normal to its faces must necessarily be nearly the same as the stress in this direction in the neighboring concrete. Provided it is granted that a thin plate embedded in concrete will accept a stress which is nearly equal to that in the surrounding concrete, it is not difficult to visualize how the plate might be converted into a stress meter. It is only necessary then that there be some means of determining the stress through the plate. Mention will be made of means which were attempted in early stages of the development, and of the means employed in the present stress meter. The early trials are offered as a background for further development of the stress meter, because although the present design appears to promise satisfactory results, better devices will undoubtedly be developed.

B. Early Designs of Stress Meters

The first stress meter consisted of mica-insulated ribbons of metal foil between sheets of steel, sealed around the edges to form a plate-like device. The stress through the device was determined by measuring the change in electrical resistance of the metal ribbons. Soft metals were preferred for the ribbons because of their relatively large variation in resistance with stress as shown in Table 1 below. A multiplication of the sensitivity, to an extent of about 5 fold, was obtained by making the area of the ribbons only a fractional part of the total.

Table 1. Change in Resistance of Metals due to Change in Pressure of One Lb. per Sq. In.

	At 0 p.s.i.	At 170,000 p.s.i.*
Indium	- .000086%	- .000103%
Tin	- .000073	- .000066
Cadmium	- .000074	- .000059
Lead	- .000100	- .000085
Zinc	- .000038	- .000029
Aluminum	- .000030	- .000025
Copper	- .000014	- .000013
Iron	- .000017	- .000015
Antimony	+ .000086	+ .000074
Bismuth	+ .000108	+ .000015

* These values presented to indicate how nearly constant resistance coefficient remains over large range of pressure.

Note:- All values are computed from data in Smithsonian Physical Tables.

The first stress meter gave only enough promise to warrant trial of several modifications. Difficulties were (1) the sealing of the edges and the design of lead connections, (2) the assurance of initial contact between steel sheets, mica, and ribbon, (3) the elimination of continued plastic flow from the ribbons, (4) the correction for temperature changes, and (5) the obtaining of sufficient sensitivity.

Several years after the first stress meter had been abandoned, Dr. P. W. Bridgman of Harvard University suggested to the writer that he use manganin ribbon and thus eliminate the need for temperature compensation. As regards the elimination of temperature troubles, the manganin is excellent, because it has almost no resistance change due to temperature. But the pressure sensitivity of manganin is low, the change in resistance being only 0.0016% for a change in pressure of 100 p.s.i. A multiplication of sensitivity of more than 100-fold would have to be accomplished to obtain sufficient sensitivity for field measurements, where long lead wires and portable test sets must be used. The local stresses in the materials of the stress meter would be likely to exceed acceptable limits if such a large multiplication were to be attempted.

The second stress meter consisted of a sheet of celluloid or bakelite between sheets of steel, with cells of granulated carbon in the celluloid or bakelite. The stress was indicated by change in resistance of the carbon cells.

It was believed that if the design gave sufficient promise, a more suitable material than the celluloid or bakelite could be found. The greatest difficulty was lack of reproducibility, presumably due to the instability of the carbon. No granular material can be expected to return exactly to its original condition after being loaded. Because of this great failing, the scheme was abandoned, despite great sensitivity.

The third stress meter employed fine steel wire embedded in a sheet of celluloid, so arranged that stress through the celluloid sheet would change the stress and resistance of the steel wire. This design showed a considerable amount of promise but failings were that the celluloid was not a suitable material because of its high thermal expansion and high Poissons ratio, proper elastic modulus could not readily be obtained, sensitivity was very low, and temperature compensation was difficult. The scheme seemed to offer promise with possible improvement but in view of the indicated large amount of necessary development, the idea was set aside.

The fourth stress meter was a fluid-filled diaphragm with a pressure-sensitive detector located in a cavity accessible to the fluid. The pressure-sensitive detector was a hollow, elastic bulb wrapped with a fine size of elastic wire. Under pressure, the bulb was compressed and the tension and resistance of the surrounding wire were reduced. The scheme seemed to have more promise than any previously tried but was set aside for future development when a more satisfactory pressure-sensitive detector might be obtained. As it was designed, with a detector consisting of a glass bulb wrapped with music wire, the sensitivity was not sufficient and provision for temperature correction was not readily made.

C. Present Form of Stress Meter

The design of stress meter that has offered a considerable amount of promise and is now being used in certain modern dams consists of a fluid-filled diaphragm, in which the pressure of the fluid deflects a small internal plate. A cross section of this stress meter is shown in Fig. 2. The essentials of operation are as follows: When the concrete surrounding a stress meter is subjected to compressive stress which has a component perpendicular to the faces of the diaphragm proper of the stress meter, the concrete presses against the faces of the diaphragm. The pressure in passing through the diaphragm, must also pass through the mercury film. The pressure, or compressive stress, is thus converted into fluid pressure in the mercury film. The fluid pressure, acting in all directions, deflects the internal plate upward, elastically. The elastic deflection of the internal plate is directly proportional to the intensity of the fluid pressure, which is in turn equal to the compressive stress applied to the diaphragm. The deflection of the internal plate is therefore an index of the compressive stress in the concrete.

The deflection of the internal plate of a stress meter is measured by means of a small elastic-wire strain meter mounted in one fact of the diaphragm. As previously stated, the strain meter contains two electrical-resistance coils (not shown), which are made by looping fine steel wire over porcelain spools, with the wire under a predetermined amount of initial tension. The mounting of the porcelain spools is such that as the deflection of the internal plate increases, steel bar "A" (see Fig. 2) moves upward and the tension and electrical resistance of the strands of one coil increase while the tension and electrical resistance of the second coil decrease. The ratio of the resistances of the two coils increases proportionately as the deflection of the internal plate increases. Because this ratio is independent of the change in resistance due to change in temperature, and because lead wire effects are largely compensating, field measurements of ratio can be made readily to an accuracy of 0.01%. This sensitivity is ample to permit stress measurements to 5 p.s.i. or less. Moreover, temperatures can be determined by means of the same coils that are used for indicating the plate deflection, because the combined resistance of the two coils varies substantially only with temperature and not with deflection.

The essential steps in the functioning of the stress meter are then as follows: (1) Compressive stress in the concrete subjects the fluid film in the diaphragm of the stress meter to a fluid pressure of an intensity substantially equal to the component of compressive stress normal to the diaphragm; (2) The fluid pressure in turn deflects the internal plate by an amount which is proportional to the intensity of the pressure; (3) The deflection of the internal plate moves a steel bar on which one end of each of two sets of tensed wires are mounted and thus increases the tension in one set of wires of the strain-meter unit and decreases the tension in the other set, both in direct proportion to the deflection; (4) The changes in tension in the two sets of elastic wires changes their electrical resistance ratio in direct proportion to the changes in tension; and (5) The change in electrical resistance ratio changes the balance reading or "observed resistance ratio" in a special Wheatstone-bridge type of testing set which is connected to the strain-meter unit by a 3-wire conductor; any changes in the observed resistance ratio denotes a change of stress in the concrete around the stress meter. When it is stated that compressive stress is "measured" by means of the electrical testing set, or even by means of the stress meter, it should be understood that the measurement is indirect to the extent noted in this paragraph. An indication of how closely the stress meter approaches the ideal functioning just outlined will be provided later in the form of analyses and of actual test results.

The strain-meter unit of the stress meter is covered with cloth to isolate it from the hardened concrete as indicated in Fig. 2. This protects the unit from being distorted by volume changes of the concrete and also prevents it from transmitting unwanted stresses to the diaphragm. Because the covered strain-meter unit has a sectional area of less than one square inch, its simulation of a void in the concrete can be shown to have little effect.

The rim of the diaphragm proper, as shown in Fig. 2, is thinner than the remainder, in order to be somewhat flexible. The rim is covered with cloth to keep it from being in close contact with the concrete. Thus, the rim, like the strain-meter unit, acts as a stress-free void in the concrete. In this way is avoided the uncertainty of an unknown amount of stress being taken by the continuous metal rim. Actually, the analyses of behavior to be presented later take account of the fact that the wrapped rim is stress-free.

The strain-meter unit contains two chambers, one of which houses the elastic steel wires, while the other is a sealing chamber for the lead-wire terminals. This duplication is purposely made in order that the chamber containing the steel wires can be kept free from all contaminating material, such as the rubber sheathing of the lead wires. Furthermore, it is not easy to make a positive seal at the point where the lead cable enters the metal case. Therefore, the sealing chamber is filled with cable joint compound, a material derived from coal tar, that is solid at ordinary temperatures. The elastic-wire chamber is filled with castor oil, which has been found to be the best liquid preventative of corrosion for the steel wires under the prevailing conditions. Besides being a preservative for the steel wires, the castor oil prevents serious temperature rise of the extremely fine wires during passage of electrical current in the measurement of their relative resistances.

All of the main parts of the strain-meter unit are of steel, including the wires, the frame, and the cover which acts as a part of the frame. The various parts then expand equally, and consequently the thermal expansion of strain meter parts is compensating. It has already been explained that the strain-meter unit is designed so as to permit measurement of resistance ratios of the steel wires instead of actual resistances and thus to be compensated also for the effect of temperature change on electrical resistance. There remains, however, a third effect of temperature for which the stress meter as a whole is not compensated, namely, the thermal expansion of the diaphragm and its fluid film. The last named thermal effect is a serious one which is discussed at length later in this thesis.

The proper embedment of a stress meter in concrete at time of casting is important, especially when the stress meter is to be placed horizontally to measure vertical stress. Changes in thickness of the diaphragm are not many millionths of an inch under most conditions and therefore the contact with the concrete must be good. The bleeding, or tendency of water to rise to the surface of concrete as solid particles settle, must be substantially zero or a water film will collect on the under side of the stress meter diaphragm before the concrete hardens. The most satisfactory procedure seems to be, before embedding the stress meter to wait until the concrete has begun to stiffen and all bleeding has stopped. A flat surface is made at the proper depth in the concrete and the bottom side of the diaphragm is "bedded" into this surface. Then the remainder of the stress meter is carefully covered with concrete and the lead-wire cable is carried to the desired terminal point or switchboard.

III. ANALYSES OF BEHAVIOR OF STRESS METERS IN CONCRETE

A. Purpose of Analyses

The behavior of a stress meter in concrete is fully defined by two actions which may be considered separately. Firstly, there is the simple action when the concrete and diaphragm tend to compress equally. Secondly, there is the effect of unequal dimensional changes due to whatever cause, or in other words, the common case where the thickness of the diaphragm tends to change by a different amount than does an equal thickness of adjacent concrete. The unequal dimensional changes may be due to a variety of causes, of which the most important are (1) unequal thermal expansion of concrete and diaphragm, (2) unequal moduli of elasticity, (3) plastic flow of concrete, and (4) shrinkage or growth of concrete. When the moduli of elasticity, or compressibilities, of diaphragm and concrete are not equal, the case can be treated as though the compressibilities are equal but that either the diaphragm or the concrete has a contraction or expansion in addition. Thus an analysis of the effect of unequal dimensional changes provides the basis for all departures from the ideal action in which the diaphragm and concrete tend to compress equally.

It has already been explained that when the application of stress causes the concrete and the stress meter diaphragm to compress equally (disregarding the effect of the wrapped rim for the present), it is inevitable that each be under the same stress. The fluid in the diaphragm then receives the full compressive stress of the concrete and the internal plate is deflected a corresponding amount. For this idealized case, the shape of the diaphragm is of no consequence because whatever the shape, the diaphragm simulates the concrete and is equivalent to the concrete it replaces as far as stresses are concerned. This suggests at once the desirability of making the compressibility of the diaphragm at least approximately equal to that of the concrete.

The shape of the stress meter becomes important, however, when differences in deformation of stress meter and concrete are involved. If the concrete shrinks, for example, some compressive stress will be applied to the diaphragm by the shrinking concrete, although there may have been no stress in the surrounding concrete. Similarly, if the concrete is more compressible than the diaphragm, then when the concrete is loaded, the extra contraction of the concrete will apply an extra compressive stress to the diaphragm. Thus, whenever the diaphragm and the concrete tend to compress unequally, there will be an error in the stress indicated by the stress meter. The amount of this error is closely related to the ratio of thickness to diameter of the stress meter diaphragm, as well as to the extent of the difference in deformation. If the diaphragm is thin, the error will be small, as will be proved below.

As a preliminary to actual analyses, an attempt is made in Fig. 3 to demonstrate the fact that only a small stress can be thrown on the stress meter diaphragm when surrounding concrete contracts. This illustration is made up of three parts, (a) a cross section of a half of the stress meter diaphragm to natural scale, (b) a partial cross section of the half diaphragm to greatly magnified vertical scale (about 7000 times) so as to portray an assumed concrete contraction, and (c) a repetition of (b) but showing also the extent of the absurdity that results if the contracting concrete is assumed to compress the diaphragm to the full extent of the contraction desired by the concrete; i.e., the amount the diaphragm would contract if it were concrete.

Fig. 3c shows why the shrinkage of concrete around a stress meter diaphragm subjects it to very little stress. The figure shows that the shrinking concrete can be "warped" around the diaphragm by a relatively small reaction from the diaphragm. This figure is based on an assumed shrinkage of 100 millionths of an inch per inch. If the diaphragm were to be compressed by this full amount, the stress through the diaphragm would necessarily be 200 p.s.i., because the effective modulus of elasticity of both diaphragm and concrete is assumed to be 2,000,000 p.s.i. But if there is 200 p.s.i. on the diaphragm, there must also be 200 p.s.i. on the concrete with which the diaphragm is assumed to be in contact. The absurdity of such a large stress on the diaphragm becomes apparent when it is shown that 200 p.s.i. is able to push the concrete away from the diaphragm by an amount of more than 500 millionths of an inch (computed according to method described later). If the concrete is to compress the diaphragm at all, its average deflection must be less than 18.7 millionths (the equivalent of 100 millionths shrinkage over the 0.187-inch half thickness of the diaphragm).

It should be clear that neither can the diaphragm be compressed by such a large force as that corresponding to the full contraction of the concrete, nor can it entirely avoid being compressed as the concrete contracts. The intermediate condition that actually results and provides both force equilibrium and continuity, is that a small fractional part of the 200 p.s.i. is thrown on the diaphragm to compress it slightly, and this same force deflects the concrete from its desired position just sufficiently to keep it in contact with the diaphragm. It is the purpose of the analyses to reveal the true conditions quantitatively for several designs of stress meters.

B. Method of Analysis

Analyses were made by a trial-and-error process to determine to just what extent stress meter diaphragms of various designs would be influenced by differences in tendencies of diaphragm and adjacent concrete to deform. An imaginary midplane through the diaphragm and extending into the surrounding concrete was assumed to exist. In view of the fact that the concrete and diaphragm were assumed to be symmetrical about this plane, it was necessary that the imaginary plane remain plane or continuity would not be preserved. The concrete and half diaphragm on one side of the plane were treated as a semi-infinite solid, with no regard for the other half of the solid except that in the end the plane must still be plane in order that the two halves fit. The concrete was assumed to contract by an amount of 100 millionths of an inch per inch, or 18.7 millionths of an inch over the 3/16 inch half thickness of the diaphragm. As a first step, it was assumed that there was no stress in the half diaphragm, in which case it protruded 18.7 millionths outward from the plane of the contracted concrete. In order to restore the plane, compressive forces were applied to the half diaphragm in an attempt to bring it into the plane of the concrete. Such forces brought the diaphragm back, but they upset force equilibrium at the same time as they distorted the surrounding concrete. Therefore tensile stress was applied outside the boundary of the diaphragm to maintain force equilibrium and to keep the concrete surface in the plane. Thus, by trial, variable compressive stresses were imposed on the diaphragm and exactly balancing tensile stresses were imposed on the surrounding concrete until the plane had been restored.

In making the analyses, advantage was taken of the fact that the forces to be applied to the half diaphragm and surrounding concrete were symmetrical about the axis of the diaphragm. For practical purposes, therefore, the plane could be considered to be made up of concentric rings of various radii, each ring to be subjected to some intensity of stress as found by trial. It was desired, then, to know the deflections that would be produced at various radii by unit intensity of stress on any ring. For convenience, a plane through one face of the diaphragm was taken instead of the midplane as the surface of the semi-infinite solid; the deformations of the half thickness of the diaphragm and of a like thickness of surrounding concrete were computed separately and added to the deflections of this plane.

The deflections of the surface of a semi-infinite solid due to stress applied over an area in the shape of a ring were obtained indirectly from equations giving the deflections due to stress applied over circular areas. The deflections due to uniform compressive stress applied over the entire area within the larger of two concentric circles, were added to the deflections (negative) due to tensile stress of equal intensity applied over the area within the smaller circle, the net result being the deflections due to a pure compressive stress over the ring area included between the two circles. Equations for the deflection of the surface of a semi-infinite solid subjected to normal load over a circular area are developed in most books on the theory of elasticity. Such an equation for the normal deflection of the surface of a semi-infinite solid having a modulus of elasticity "E" and a Poissons ratio of m, due to normal stress of intensity "q" applied over a circular area of radius "a" is as follows for any distance "r" from the center and <u>OUTSIDE</u> the loaded area:

$$\text{Deflection} = (1-m^2)\frac{4qr}{\pi E}\int_0^{\frac{\pi}{2}} \overset{\text{(E)}}{\sqrt{1-\frac{a^2}{r^2}\sin^2\theta}}\, d\theta \; -$$

$$-\frac{4qr\,(1-m^2)}{\pi E}\left(1-\frac{a^2}{r^2}\right)\int_0^{\frac{\pi}{2}} \overset{\text{(K)}}{\frac{d\theta}{\sqrt{1-\frac{a^2}{r^2}\sin^2\theta}}}$$

in which θ is the angle between (1) a line through both the point where the deflection is desired and through the elemental area, and (2) the radius through the point where this line strikes the circle. The terms of the above equation can be expressed in any convenient units; for engineering purposes, it is convenient to adopt units of pounds, inches, and radians for force, length, and angle, respectively.

The solution of the above equation and applying only to the area OUTSIDE the loaded circle is as follows:

$$\text{Defl.} = \frac{2qr}{E}(1-m^2)\left\{1 - \left(\frac{1}{2}\right)^2 \frac{a^2}{r^2} - \left(\frac{1}{2}\cdot\frac{3}{4}\right)^2 \frac{a^4}{3r^4} \left(\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\right)^2 \frac{a^6}{5r^6} - \ldots\right\}$$

$$- \frac{2qr}{E}(1-m^2)\left\{1-\frac{a^2}{r^2}\right\}\left\{1+\left(\frac{1}{2}\right)^2 \frac{a^2}{r^2} + \left(\frac{1}{2}\cdot\frac{3}{4}\right)^2 \frac{a^4}{r^4} + \left(\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\right) \frac{a^6}{r^6} + \ldots\right\}$$

Inside the loaded area, the corresponding equation is

$$\text{Deflection} = \frac{4qa}{\pi E}(1-m^2)\int_0^{\frac{\pi}{2}} \sqrt{1 - \frac{r^2}{a^2}\sin^2\phi}\; d\phi$$

in which $\phi = \frac{\pi}{2} - \theta$

The corresponding series solution for this equation applying only to the area INSIDE the loaded circle is

$$\text{Defl.} = \frac{2qa}{E}(1-m^2)\left\{1 - \left(\frac{1}{2}\right)^2 \frac{r^2}{a^2} + \left(\frac{1}{2}\cdot\frac{3}{4}\right)^2 \frac{r^4}{a^4} + \left(\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\right) \frac{r^6}{a^6} + \ldots\right\}$$

In computing the deflections for points near the boundary of a loaded area, where "r" and "a" are nearly equal, the above series converge very slowly and the extension to even as many as 20 terms does not provide sufficient accuracy for determining deflections due to loaded rings. Therefore, an attempt was made to differentiate the equations for deflections due to load applied over a circular area so as to obtain directly the deflections due to load applied over a ring area. In the trials which were made, no advantage resulted, but it is believed that a better solution is possible and that a good problem is here provided for a mathematics student.

For the present purpose, a satisfactory solution of the problem of determining accurate deflections due to load applied over a ring of any limiting radii was obtained by making use of the dimensionless character of the main portions of the above equations. A single set of curves was developed from which the deflections due to load on rings of any size or shape could be determined, all as explained below.

The equations for deflections due to load on circular areas were written as follows:

$$\Delta = \frac{4\ qr}{\pi E}(1-m^2)\left[A - \left(1 - \frac{a^2}{r^2}\right)K\right] = \frac{2qr\ F}{E}(1-m^2) \quad \text{(outside loaded area)}$$

$$\Delta = \frac{4\ qa}{\pi E}(1-m^2)\ A = \frac{2qa\ F}{E}(1-m^2) \qquad \text{(inside loaded area)}$$

in which equations, A and K are the elliptic integrals as follows:

$$A = \int_0^{\frac{\pi}{2}} \sqrt{1 - \frac{a^2}{r^2}\sin^2\theta}\ d\theta \quad \text{or} \quad \int_0^{\frac{\pi}{2}} \sqrt{1 - \frac{r^2}{a^2}\sin^2\phi}\ d\phi$$

$$K = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1 - \frac{a^2}{r^2}\sin^2\theta}}$$

while F is a variable factor having a different value for each value of $\frac{a}{r}$ or $\frac{r}{a}$ as defined by the equations.

Values of the factor "F" were computed for various values of "a/r" and of "r/a" from 0 to 1. In all, 180 values were computed, corresponding to the ninety values each of the elliptic integrals "A" and "K", which are available to 5 significant figures in Peirce, "A Short Table of Integrals." The integral "A" is designated "E" by Peirce but it is changed to "A" here to avoid confusion with the symbol for modulus of elasticity. Only the ninety factors corresponding to the available values of the elliptical integrals were used, because the computation of additional factors to sufficient accuracy by means of the series solutions would have been laborious and unnecessary. Poissons ratio was assumed to be zero in computing the factor "F" because the 2.5 per cent difference (.16 squared) resulting from its inclusion was not considered to be significant here.

The 180 factors are presented in Table 2. In preparation for computing factors for ring areas, the factors in Table 2 for circular areas were plotted to large scale and curves were drawn to permit intermediate factors to be read to equal accuracy. It is important to note that the factors apply for any radius of loaded circle, as well as for any loading intensity and any modulus of elasticity. In view of the fact that the plotted curves of factors for circular areas are quite numerous and are only an intermediate step, they are not included in this report. Table 2 contains all the basic data of this step.

From the factors for circular areas, factors for ring areas were computed by difference. A complete series of factors was obtained for rings covering a wide range in slenderness, which is here defined as the ratio of ring width to average radius. Thus again, the factors were kept independent

Table 2. Factors for Determining Deflections due to Normal Load over Circular Area on Surface of Semi Infinite Solid. (See equation below)

Ratio of Radii[1]	Factors F		Ratio of Radii	Factors F		Ratio of Radii	Factors F	
	Outer[2]	Inner[3]		Outer[2]	Inner[3]		Outer[2]	Inner[3]
.0175	.0002		.5150	.1374		.8746	.4380	.7650
.0349	.0006	.9998	.5299	.1459	.9257	.8829	.4482	.7590
.0523	.0013		.5446	.1546		.8910	.4584	.7529
.0698	.0025	.9989	.5592	.1634	.9166	.8988	.4685	.7470
.0872	.0038		.5736	.1724	.9119	.9063	.4784	.7410
.1045	.0055	.9973	.5878	.1815	.9071	.9135	.4881	.7351
.1219	.0074		.6018	.1907	.9022	.9205	.4977	.7293
.1392	.0097	.9952	.6157	.2001	.8972	.9272	.5071	.7234
.1564	.0123		.6293	.2096	.8922	.9336	.5164	.7177
.1736	.0152	.9925	.6428	.2194	.8870	.9397	.5254	.7121
.1908	.0184		.6561	.2294	.8817	.9455	.5342	.7065
.2079	.0218	.9892	.6691	.2391	.8764	.9511	.5431	.7011
.2250	.0254		.6820	.2492	.8710	.9563	.5519	.6957
.2419	.0296	.9853	.6947	.2595	.8655	.9613	.5595	.6905
.2588	.0338		.7071	.2697	.8599	.9659	.5672	.6853
.2756	.0394	.9808	.7193	.2800	.8543	.9703	.5748	.6804
.2924	.0432		.7314	.2906	.8486	.9744	.5822	.6756
.3090	.0483	.9758	.7431	.3007	.8428	.9781	.5888	.6710
.3256	.0537		.7547	.3114	.8370	.9816	.5953	.6665
.3420	.0594	.9702	.7660	.3219	.8312	.9848	.6015	.6622
.3584	.0654		.7771	.3325	.8253	.9877	.6075	.6585
.3746	.0714	.9640	.7880	.3433	.8194	.9903	.6130	.6544
.3907	.0779		.7986	.3539	.8134	.9925	.6174	.6509
.4067	.0845	.9574	.8090	.3657	.8074	.9945	.6220	.6477
.4226	.0915		.8192	.3753	.8014	.9962	.6262	.6448
.4384	.0986	.9502	.8290	.3860	.7953	.9976	.6297	.6422
.4540	.1059		.8387	.3964	.7893	.9986	.6323	.6401
.4695	.1135	.9425	.8480	.4067	.7832	.9994	.6346	.6384
.4848	.1213		.8572	.4175	.7771	.9999	.6359	.6372
.5000	.1293	.9343	.8660	.4278	.7711	1.0000	.6366	.6366

Deflection = $\frac{2qr}{E} F$ (outside loaded area) or $\frac{2qa}{E} F$ (inside loaded area)

a = radius of loaded circle, r = distance from center of circle to point where deflection is desired, q = load per unit area, E = modulus of elasticity.

[1] "Ratio of Radii" represents location, expressed as a/r or r/a, outside or inside loaded circle, respectively.

[2] Outside circular area over which load is applied.

[3] Inside circular area over which load is applied.

of actual dimensions, in order that one set of curves would permit the solution for rings of any size or slenderness. The deflection factors for loaded rings are presented as a series of curves in Figs. 4 and 5.

With the help of the curves in Figs. 4 and 5, the analyses were made of embedded stress meters to determine the effect of unequal tendencies of diaphragm and concrete to contract. The method of trial and error which was used is closely analogous to the trial-load method of analysis used for designing an arch dam. In the present instance, the deflections of diaphragm and adjoining concrete are made to coincide while maintaining force equilibrium, and in the analysis of an arch dam the deflections of horizontal and vertical elements are made to coincide at common points, also maintaining force equilibrium.

It has already been explained that when the diaphragm and the surrounding concrete tend to compress equally, the stress through the diaphragm will be equal to that in the surrounding concrete. But if the concrete tends to compress more or less than the diaphragm, the stress through the diaphragm will be slightly different. A single analysis, assuming the concrete to contract while the diaphragm tends to remain constant in thickness, serves for a variety of cases.

For example, if the diaphragm has a compressibility, or modulus of elasticity, which is greater than that of the concrete, so that the concrete tends to compress more under load, the case can be treated as though the compressibilities were equal but the concrete had a contraction in addition. An analysis of the effect of this contraction will show how much different will be the stress indicated by the stress meter. Similarly, the same analysis will apply if the concrete tends to deform more or less than the diaphragm because of changes in temperature. In fact, the analysis applies for any case where the concrete tends to deform differently from the diaphragm.

The performance of a stress meter composed of a diaphragm of given shape and rigidity can best be expressed by the coined term "independence factor," which defines the degree to which the device is independent of extraneous deformation. The independence factor used to express the performance of a given stress meter for the general case when concrete and diaphragm tend to deform unequally is arbitrarily taken as the ratio of the intensity of stress imposed on the diaphragm to the stress which would have been imposed if the diaphragm had been compressed in thickness to the full amount of the contraction of the concrete. Thus, if a stress meter were to have an independence factor of 0.1 and its diaphragm were to have an effective modulus of elasticity equal to that of the concrete, the diaphragm would be compressed only one-tenth the amount of any extra contraction of the concrete. As a more explicit example, if stress were to be applied to concrete containing such a stress meter and if the concrete were to have a modulus of elasticity 10 per cent different from that of the diaphragm, the error in indicated stress would be only about 1 per cent. According to the definition, the lower the independence factor, the better the stress meter.

In order to simplify the determination of independence factors, the steel plates comprising the diaphragm of the stress meter are assumed to be either completely flexible or completely inflexible. No intermediate conditions need be considered, because it will be shown that the resistance of the diaphragm to bending does not greatly affect the independence factor.

The conditions at the rim of a stress meter diaphragm have already been mentioned, but they will be recalled before results of analyses are discussed. If the welded rim were left bare when a diaphragm were cast in concrete, an unknown amount of stress would be transmitted directly through the solid-metal rim. In early stress meters, expensive slots were provided in relatively thick diaphragms so as to transfer load from the rim to the mercury film some distance in from the edge. This design was so unsatisfactory that a simpler design was adopted for which the feature was a fabric-wrapped rim. This wrapping produces a void area as far as stress is concerned, and the stress that would normally fall on this area is shared about equally by the diaphragm and surrounding concrete; only a small stress escapes around the metal rim. This makes the effective area of the diaphragm somewhat greater than the net area in contact with concrete, and tests indicate that to include half of the rim area with the diaphragm is approximately correct.

C. Results of Analyses

It is recalled again that no analyses are necessary for the simple case when a stress meter tends to contract under load exactly as much as the surrounding concrete. The stress meter must then indicate the true compressive stress, provided the calibration is accurate. Analyses are necessary only for the purpose of determining the extent of error when concrete and diaphragm tend to compress unequally.

In Fig. 6 are shown the conditions which result when a stress meter tends to contract by a different amount from the surrounding concrete. Because, as previously explained, only the effect of a difference in desired deformation need be analyzed, the diaphragm is assumed to have no desired deformation, while the concrete contracts by a nominal 100 millionths of an inch per inch. The results in Fig. 6 are for a stress-meter diaphragm which is completely flexible. The upper two diagrams are quarter sections of a stress-meter diaphragm (strain-meter unit omitted) before and after the contraction of the concrete. The vertical scale is greatly enlarged so as to show the deformations; for this reason the section has been cut to confine attention mainly to the boundary of the diaphragm that is in contact with the concrete. The space marked "void" represents the wrapped rim, which cannot accept stress from the concrete.

A comparison of the two uppermost diagrams in Fig. 6 shows that the concrete upon contracting has "warped" around the stress-meter diaphragm and has bent it without having compressed it noticeably. The third diagram, near the bottom of Fig. 6, shows the forces which are consistent with the deformations. No credence should be given to the apparent discontinuity of the stress distribution in the surrounding concrete, as the method of analysis necessitates the assumption of uniform stress on each ring of finite width. The more refined the analysis, the closer the stress distribution would approach a smooth curve. The stress over the face of the diaphragm is necessarily uniform in this case, because the diaphragm is assumed to be flexible and it contains a fluid film.

It may be noted that the compressive stress on the diaphragm is only 7.8 p.s.i., whereas if the diaphragm had been compressed to the full extent of the concrete contraction, the stress would have been 200 p.s.i. (E = 2,000,000 p.s.i.). Thus, the independence factor is 0.039 (= 7.8/200), in accordance with the definition previously stated. The total compressive stress on the diaphragm is exactly balanced by tension in the surrounding concrete as is required by the conditions of the analysis.

In Fig. 7 are the graphical results of an analysis for an inflexible diaphragm. In this case the compressive stress on the diaphragm need not be uniform, but the diaphragm must remain substantially plane. The variation from planeness reflects the inexact nature of the analysis. Again, the diaphragm is compressed by only a fractional amount of the concrete contraction. The average stress on the diaphragm amounts to 9.0 p.s.i., which may be compared again with the 200 p.s.i. corresponding to full contraction of the concrete. The independence factor in this case is 0.045. Comparing the latter factor with the factor of 0.039 as found above for a flexible diaphragm, it may be seen that the difference is not great. For practical purposes, the average of the two factors is considered to be sufficiently accurate for an actual stress meter, whose diaphragm is somewhat flexible.

A sample computation sheet is presented as Table 3 to show the origin of such results as are included in Figs. 6 and 7. The data in Table 3 comprise one final trial to determine the stress distribution on and around a particular stress meter due to a contraction of the concrete, according to the method explained above. The concrete is assumed to contract 100 millionths of an inch per inch while the diaphragm tends to remain at fixed thickness. The final stresses must be such that the entire area of the diaphragm in contact with the concrete is deflected relatively to the surrounding concrete by 100 millionths of an inch per inch, or 18.7 millionths in the half thickness of the diaphragm. The data can best be visualized by assuming as a first step that a plane is cut through the concrete and the diaphragm midway between the two faces of the diaphragm. After the concrete contraction, the diaphragm will extend out 18.7 millionths of an inch from the plane of the concrete, because in the cut condition there can be no compression on the diaphragm. The stresses shown in Table 3 are such as to deflect the diaphragm back into the plane of the concrete and yet to maintain the concrete plane and to leave no unbalanced forces. The deflections are actually reckoned from the plane of the diaphragm face instead of the midplane and the additional deflections in the half

TABLE 3. EXAMPLE OF TRIAL ANALYSIS TO DETERMINE EFFECT OF CONCRETE CONTRACTION UPON INFLEXIBLE STRESS METER OF 3.4-INCH NET RADIUS.

Stress Intensity (p.s.i.)	Ring Radii	DEFLECTIONS AT VARIOUS DISTANCES FROM CENTER, millionths inch											
		0	1.0	2.0	3.0	3.2	3.4	3.8	4.0	4.5	5.0	6.0	7.0
				(DUE TO COMPRESSION ON STRESS METER)									
5	0.00- 1.94	9.7	9.0	5.8	3.4	3.2	3.0	2.5	2.3	2.1	1.9	1.6	1.4
8	1.94- 2.37	3.4	3.7	5.7	3.0	2.7	2.5	2.2	2.1	1.7	1.5	1.3	1.1
10	2.37- 2.90	5.3	5.5	6.5	6.8	5.7	5.1	4.4	4.1	3.5	3.1	2.4	2.0
13.5	2.90- 3.20	4.1	4.2	4.7	7.9	6.9	5.3	4.2	3.9	3.3	2.9	2.3	1.9
28	3.20- 3.37	4.7	4.8	5.2	6.9	9.2	8.3	5.4	5.0	4.2	3.1	2.8	2.3
103	3.37- 3.40	3.5	3.6	3.9	4.9	5.6	8.5	4.4	4.0	3.2	2.8	2.2	1.8
Total positive deflection		30.7	30.8	31.8	32.9	33.3	32.7	23.1	21.4	18.0	15.3	12.6	10.5
				(DUE TO TENSION IN SURROUNDING CONCRETE)									
35	3.80- 3.84	1.3	1.4	1.4	1.7	1.8	1.9	3.5	2.1	1.5	1.3	1.0	0.8
15	3.84- 4.03	2.9	3.0	3.2	3.6	3.8	4.0	5.5	6.0	3.5	2.9	2.3	1.8
10	4.03- 4.46	4.3	4.3	4.6	5.0	5.2	5.4	6.1	7.1	6.8	4.8	3.7	3.0
2.2	4.46- 5.45	2.2	2.2	2.3	2.4	2.5	2.5	2.7	2.8	3.5	3.7	2.4	1.9
0.35	5.45- 8.15	0.9	0.9	1.0	1.0	1.0	1.0	1.0	1.0	1.1	1.2	1.4	1.4
0.2	8.15-12.2	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.9	0.9	0.9	1.0
0.1	12.2 -18.3	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
Total negative deflection		13.0	13.2	13.9	15.1	15.7	16.2	20.2	20.4	17.9	15.4	12.3	10.5
Net deflection		17.7	17.6	17.9	17.8	17.6	16.5	2.9	1.0	0.1	-0.1	0.3	0.0
Direct Strain Correction		1.0	1.0	1.0	1.0	1.0	1.0	-3.3	-1.4	-0.2	-0.2	0	0
Final deflection		18.7	18.6	18.9	18.8	18.6	17.5	-0.4	-0.4	-0.1	-0.3	0.3	0.0

SUMMARY:-

Contraction of concrete assumed 100 millionths per unit, or 18.7 millionths inch for the half thickness of diaphragm.

Stress which would result if diaphragm were compressed full amount is 100 millionths multiplied by the assumed "E" of 2,000,000 p.s.i., or <u>200</u> p.s.i.

Stress found by the analysis above is 441 lb. total compression on diaphragm divided by the effective area of 40.8 inches, or <u>10.8</u> p.s.i.

Independence factor is then 10.8 divided by 200, or <u>0.054</u>.

(Total tension in surrounding concrete is approximately 440 lb.)

NOTE:-

"Direct Strain Correction" is the adjustment to allow for the strain between the plane of the diaphragm face and the midplane of the diaphragm.

thickness of the diaphragm are added under the label "Direct strain correction." It will be noted that the deflections over the diaphragm are substantially 18.7 millionths inch as required and that the concrete is maintained substantially undeflected.

In the case of a flexible diaphragm, the procedure is the same as for the inflexible diaphragm represented by Table 3 except that the deflection need not be uniform over the diaphragm. Instead, the stress on the diaphragm must be uniform and the weighted average deflection must be 18.7 millionths inch.

Results of a few analyses of stress meters having the approximate dimensions of those now in use are presented in Table 4 below. Among the results shown, the best independence factor of about 0.04 is indicated for a stress meter having a wide wrapped rim (0.8 in.). For such a stress meter, the modulus of elasticity of concrete and diaphragm could differ by about 25 per cent without causing an error of more than 1 per cent. But the wide stress-free rim introduces a considerable amount of uncertainty as to how much stress goes to the diaphragm and how much to the surrounding concrete. A more satisfactory diaphragm is the one with a wrapped rim 0.4 inches wide, for which the independence factor is about 0.05, which is as low as need be. The diaphragm with a stress-free rim only 0.1 inch wide has a sufficiently low independence factor, but with such a small allowance it is impracticable to avoid passage of an appreciable amount of stress through the welded rim.

Table 4. Independence Factors for Stress Meters of Varying Peripheral Widths

Net Diaphragm Radius	Effective Area	Width of Stress-Free Rim	Independence Factors	
			Inflexible Diaphragm	Flexible Diaphragm
3.4 in.	45.8	0.8 in.	0.045	0.038
3.4 in.	40.9	0.4 in.	0.054	0.045
3.7 in.	44.2	0.1 in.	0.076	0.051

Note:- All diaphragms 3/8 inch thick and of same compressibility as concrete.

IV. DESIGN CONSIDERATIONS

A. Sensitivity and Range as Governed by Internal Plate

The sensitivity of a stress meter can be varied almost at will in the design, but account must be taken of the relation between sensitivity and range, and of the resulting compressibility. Among the primary factors to be considered in the design are the dimensions and maximum stresses of the internal plate, and the composition of the metal or alloy to be used. Obviously these quantities are related. For example, if the dimensions of the plate are such as to cause high stress, a metal or alloy with a corresponding elastic limit must be employed. Curves showing the relation of (1) maximum stress, (2) deflection due to a nominal load, (3) thickness, and (4) radius, all for a plate clamped at the rim, are presented in Fig. 8. The curves are not precise, nor need they be, because not only is there always some doubt about the fixity of the rim of such a plate, but every stress meter is individually calibrated.

In the present stress meters, the internal plate has a radius of 0.75 inch and a thickness of 0.08 inch. This results in a flexural stress at the rim of about 40,000 p.s.i. for an applied load of 600 p.s.i., which is the ordinary range. A good grade of carbon steel serves very well for this set of conditions. The resulting deflection for 600 p.s.i. is about 0.003 inch, and the least reading is about 5 p.s.i.

B. Effect of Temperature on the Stress Meter

One of the most difficult problems in the design of a stress meter is to secure substantial independence from the effect of temperature changes. The problem is not with the strain-meter unit which detects the deflection of the internal plate, because this unit is compensated for temperature effects. It has been shown by analysis as well as by actual test that as long as the essential members of the strain-meter unit are all of steel there is no sensible effect of temperature on the accurate detection of deflection. Instead, the problem is to secure a composite diaphragm which has approximately the same coefficient of expansion with respect to thickness as does an equal thickness of concrete. It appears to be essential that the diaphragm contain a film of fluid, and high thermal expansion is a characteristic of nearly all fluids, as compared with solids. In the following table, the cubical expansion of steel is compared with that of each of several fluids which might be considered for use in stress meters.

Material	Cubical Expansion per Degree F.	
	at 70° F.	at 100° F.
Steel	20×10^{-6}	20×10^{-6}
Mercury	101	101
Glycerine	281	289
Petroleum	532	558
Water	120	205

At first thought, the expansion of a thin film of mercury would not seem to be the cause of any alarm, because the coefficient is only five times that of steel. In fact, for the early stress meters only slight attention was paid to the question of temperature effects, it being believed that if the mercury film were reasonably thin there would be no trouble.

But the expansion of the fluid film in the stress meter is mostly confined to the one direction. As the temperature rises and the materials expand, only 13.3 per cent of the volume expansion of the mercury is permitted laterally; the remainder must take place normal to the diaphragm. Thus, there results an effective linear expansion of 87 millionths of an inch per inch per degree F. tending to change the thickness of the mercury film. This value is 13 times the linear expansion of the steel, instead of only 5 times. The importance of reducing the film thickness to a minimum is obvious.

The magnitude of the correction which must be applied to indicated stress to obtain true stress will depend upon several factors. The lowest correction results under the following conditions:

1. Stress meter having a minimum thickness of mercury film,

2. Stress meter having a low independence factor,

3. Concrete having a high thermal expansion, and

4. Concrete having a low modulus of elasticity.

A simple equation can be written for the tentative correction by expressing the fact that the correction in pounds per square inch is equal to the product of (1) the excess expansion of the diaphragm thickness above that of the concrete, (2) the independence factor, and (3) the modulus of elasticity of the concrete. The resulting equation is included in Fig. 9. The equation is also presented as a series of diagrams in the same figure.

A comparison of the temperature corrections for early stress meters and for those recently constructed is useful to demonstrate the progress that has been made as well as to show the magnitude of the correction under specific conditions. The thickness of the mercury film in early stress meters was about 0.125 times the diaphragm thickness, as compared with about 0.06 for recent meters. The performance factor for early meters was about 0.10 as compared with 0.05 for recent meters. Assuming an average concrete with a thermal expansion of 6 millionths per degree F., the following corrections are found from Fig. 9:

	E = 2,000,000 p.s.i.	E = 4,000,000 p.s.i.
Corrections for early stress meters	2.1 p.s.i./1° F.	4.2 p.s.i./1° F.
Corrections for recent stress meters	0.5 " "	1.0 " "

The correction is not negligible even for the recent stress meters, especially if the modulus of elasticity of the concrete is high. However, the effect of temperature is less than one-fourth of that which prevailed in early stress meters, and further improvement is now assured through a further reduction in thickness of the mercury film. The thinness of the mercury film has been limited by the inability of welding the rim of the diaphragm without some subsequent buckling of the diaphragm plates. The welding technique has gradually been improved to provide continuous films of less than 0.02 inch thickness.

The temperature correction of a stress meter is directly proportional to the modulus of elasticity of the concrete. Thus, the correction is greater for quick changes in temperature because of the absence of plastic flow, which would operate to reduce the effective modulus of elasticity.

C. Considerations Regarding Modulus of Elasticity

The effect of a difference in the compressibility of the stress meter and that of the adjacent concrete can be found by applying the independence factor to the difference in desired contraction under load. The following equations express the simple relations,

$$\text{Difference in desired contraction} = \frac{f_c}{E_c} - \frac{f_c}{E_{SM}}$$

where f_c is compressive stress in concrete in the direction perpendicular to the plane of the diaphragm, E_c is the modulus of elasticity of the concrete, and E_{SM} is the effective modulus of elasticity of the stress meter.

It follows that,

$$\text{Stress error} = \left\{ \frac{f_c}{E_c} - \frac{f_c}{E_{SM}} \right\} E_c F = F f_c \left\{ 1 - \frac{E_c}{E_{SM}} \right\}$$

where F is the inedpendence factor.

The above equations are not exact, because the independence factor is developed on the assumption that compressibility of concrete and diaphragm are equal. Actually, the compressibility of the diaphragm alters the independence factor only slightly, so long as the compressibility of the diaphragm is not more than twice that of the concrete. Therefore, the above equations can be used for all diaphragms whose compressibilities lie within the great range from infinite rigidity on the one hand to one half of the rigidity of the concrete on the other.

Whenever the diaphragm is more rigid than the concrete, the greatest error that can possibly be attributed to difference in modulus of elasticity is the full product of concrete stress and performance factor. This maximum error occurs when the diaphragm has infinite rigidity. It amounts to 5 per cent for recent stress meters having performance factors of 0.05. This is an important fact where modulus of elasticity is unknown or indefinite. For example, if the stress meter should be applied to a soil instead of concrete, and the soil were to have a compressibility of 100 times (10,000 per cent) its assumed value, the error could nevertheless be limited to 5 per cent or less.

When the diaphragm is less rigid than the concrete, the greatest error can conceivably reach 100 per cent. In other words, if the diaphragm can be compressed as much as the surrounding concrete by negligible forces, no stress would be indicated. Whenever the modulus of elasticity of the concrete is in doubt, therefore, the chance of large error is lessened by holding the compressibility of the diaphragm on the low side.

The effective compressibility of a stress meter diaphragm is the result of a combination of causes. The contribution due to direct compression of the steel is small compared with that due to "bulging" of the parts not in close contact with the concrete. The compressibility of the mercury is also a minor factor. The effective compressibility may be determined by evaluating separately the reduction in thickness of the diaphragm due to each cause. For example, the following computations apply specifically to one of the recent stress meters:

Assumptions: (a) thickness of diaphragm .375 inch, of which 0.35 is steel and 0.025 is mercury, (b) diameter of internal plate 1.5 inch and thickness such as to deflect 0.0007 inch due to uniform pressure of 100 p.s.i., (c) radius of diaphragm 3.6 inches, of which outer 3/8 inch has a wall thickness of only 1/16 inch and is not in contact with the concrete.

Results: The reduction in thickness of the diaphragm as a whole due to a stress of 100 p.s.i. is as follows:

1. Due to direct compression of steel	1.2	millionths	inch
2. Due to compression of mercury	0.6	"	"
3. Due to bulge of outer rim	4.4	"	"
4. Due to bulge of internal plate (ave. defl. 0.42 of max.)	12.7	"	"
Total	18.9	"	"

The compressibility per inch of thickness is then 18.9 divided by the thickness (3/8 inch) or 50 millionths, and the effective modulus of elasticity is then 2 million pounds per square inch. This happens to be a good average value for ordinary concrete. True, the usual quick-loading tests give higher values, but the value that governs here is the "sustained" modulus of elasticity, which includes the plastic flow due to continued load.

V. ACTUAL PERFORMANCE OF STRESS METERS

Results from stress meters in service concrete are not yet complete enough to prove the dependability of the device as now constructed. Field measurements to date, however, appear to be reasonable.

Stress meters embedded in laboratory specimens of concrete have consistently shown good results except under temperature change. As discussed above, the temperature correction has been especially high in early stress meters. The trouble is magnified by the fact that concrete usually hardens at an elevated temperature, especially when cast in large masses. The mercury film in the diaphragm is then expanded, with the result that later cooling frees the diaphragm from the concrete. An appreciable initial compressive stress is then required before the stress meter begins to operate. This type of behavior is demonstrated in Fig. 10.

The diagrams in Fig. 10 represent actual observations on two stress meters embedded in large concrete cylinders. One of the stress meters was constructed in 1935, before the magnitude of the temperature correction was appreciated, and the other was constructed in 1938. In both cases the concrete hardened at an elevated temperature. Loads were later applied with specimens at room temperature. The initial compressive stress required to bring each stress meter into operation is an indication of the magnitude of the temperature correction. It may be noted that the error in the early meter is about 180 p.s.i. as compared with about 20 p.s.i. for the recent meter. Detailed consideration of these diagrams is not warranted because, although temperature histories were recorded, there was no accurate knowledge of the elastic and plastic properties of the concretes, nor of their thermal expansions. Suffice it to say that the diagrams indicate the successful reduction in temperature correction that has been achieved in recent stress meters.

The performance of a recent stress meter embedded in a concrete specimen and loaded at constant temperature is shown in Fig. 11. In this figure are two charts, the upper one showing indicated stress plotted against applied stress, and the lower one showing measured strain also plotted against applied stress. The specimen was a 15-inch diameter cylinder of concrete containing one of the recent stress meters on its axis and containing also a strain meter placed alongside the stress meter parallel to the axis. The cylinder was loaded when the concrete was only four days old, and again when the age was 143 days. During this interval of time the concrete increased noticeably in rigidity, as revealed by the stress-strain curves in the lower chart.

In fact the strain for 500 p.s.i. was 40 per cent greater at 4 days than at 143 days. The indicated stress, however, was substantially the same at both ages, and it agreed well with the actual applied stress at both ages. Furthermore, the stress meter indicated the true stress whether the load was increasing or decreasing, despite the fact that the strains were widely different. This is believed to be the most convincing proof that the stress meter indicates compressive stress largely independent of the magnitude of the accompanying strain.

In conclusion, it should be stated that the results shown in Fig. 11 represent tests on only one of many stress meters embedded in concrete cylinders and tested similarly. The earliest such test on a stress meter of the general type now in use was made in 1932. In general, there was a gradual improvement and each successive test showed a closer approach to the goal of independence from extraneous deformations. The results shown in Fig. 11 are the best that have been obtained thus far, but further improvement in the stress meter is anticipated, especially in the correction due to temperature change.