The in-phase real (Z_r) component and the out-of-phase imaginary (Z_i) are given by (see main text):

$$Z_r = R_c + \frac{R_p}{1 + \omega^2 R_p^2 C_{dl}^2} \quad (1)$$

and

$$Z_i = -\frac{\omega R_p^2 C_{dl}}{1 + \omega^2 R_p^2 C_{dl}^2} \quad (2)$$

From Equation 1:

$$1 + \omega^2 R_p^2 C_{dl}^2 = \frac{R_p}{Z_r - R_c} \quad (3)$$

or

$$\omega R_p C_{dl} = \sqrt{\frac{R_p}{Z_r - R_c} - 1} \quad (4)$$

Substituting Equations (3) and (4) into Equation (2)

$$Z_i = -\frac{R_p \sqrt{\frac{R_p}{Z_r - R_c} - 1}}{\frac{R_p}{Z_r - R_c}} \quad (5)$$

or,

$$Z_i = -\sqrt{\frac{R_p}{Z_r - R_c} - 1}\,(Z_r - R_c) \quad (6)$$

Squaring both sides of Equation (6)

$$Z_i^2 = \left(\frac{R_p}{Z_r - R_c} - 1\right)(Z_r - R_c)^2 \quad (7)$$

or,

$$Z_i^2 = R_p (Z_r - R_c) - (Z_r - R_c)^2 \quad (8)$$

which can be rearranged as:

$$Z_i^2 + \left[Z_r - R_c - \frac{R_p}{2}\right]^2 - \left(\frac{R_p}{2}\right)^2 = 0 \quad (9)$$

Therefore,

$$\left[Z_r - \left(R_c + \frac{R_p}{2}\right)\right]^2 + Z_i^2 = \left(\frac{R_p}{2}\right)^2 \quad (10)$$

as given in the main text.