Algorithms for Microwave Imaging[1]

Consider a perfectly conducting cylindrical object embedded in a dissipative medium, $\mathcal{D}$, (permittivity ε and conductivity σ) and illuminated by a harmonic incident field, $\{E^i, H^i\}$, with angle of incidence, θ, with respect to y-axis (Fig. 1). The time-factor is given by $e^{-j\omega t}$. The incident electric field is linearly polarized along the z-axis. The scattered field, E^s, has only a z component and is generated by an electric surface current, J_s, on the object .

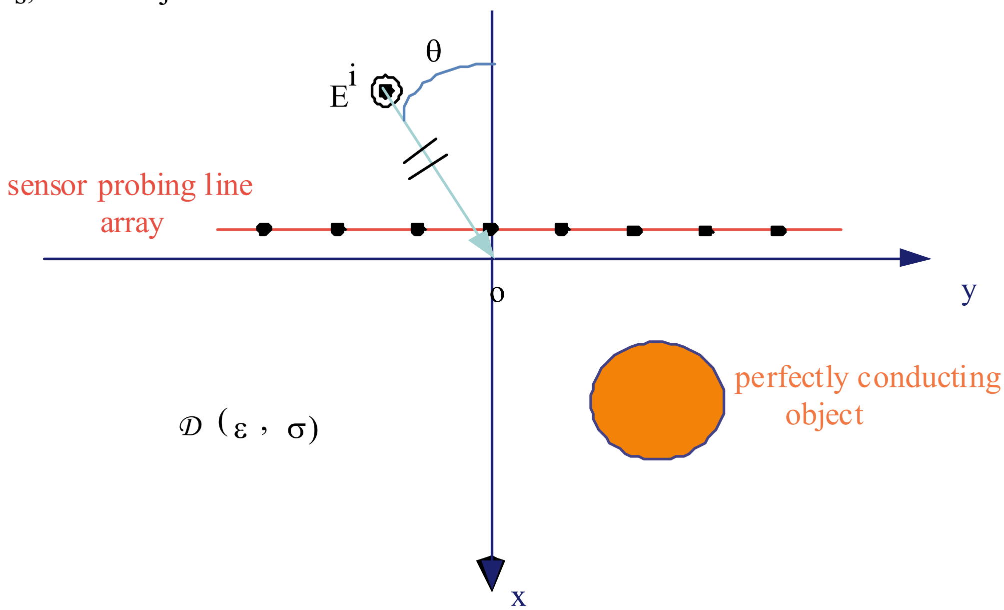

Figure 1 - 2D geometry of the problem.

The scattered field satisfies the Helmholtz equation

$$\Delta E^s + k^2 E^s = -i\omega\mu_0 J_s \qquad (1)$$

with $k^2 = \omega^2 \varepsilon \mu_0$

and is given through an integral representation

$$E^s(\mathbf{r}) = i\omega\mu_0 \int_S J_s(\mathbf{r}')\, G(\mathbf{r},\mathbf{r}')\, d\mathbf{r}' \qquad (2)$$

with $\mathbf{r}=(x,y)$ $\quad \mathbf{r}'=(x',y')$

and with the Green's function

$$G(\mathbf{r},\mathbf{r}') = i/4\, H_0^{(1)}(k|\mathbf{r}-\mathbf{r}'|) \qquad (3)$$

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[1] This summary is based on P.J.M. Monteiro, C.Y.Pichot and K. Belkebir, Computer Tomography of Reinforced Concrete (with), Chapter 12, Materials Science of Concrete, American Ceramics Society (1998).

where $H_1^{(0)}$ is the Hankel function of the first kind of order zero

Diffraction Tomography formulation

The normalized surface current , K(x,y), is given by

$$K(x,y) = \frac{J_s(x,y)}{E^i(x,y)} \qquad (4)$$

with

$$E^i(x, y) = e^{ik(y\cos\theta + x\sin\theta)} \qquad (5)$$

The Fourier transform of the Green's function is given by

$$G(x, y; x', y') = \frac{i}{2\tilde{\gamma}} \int_{-\infty}^{+\infty} e^{i\tilde{\gamma}|y - y'|} e^{2i\pi\nu(x - x')} d\nu \qquad (6)$$

$$\tilde{\gamma}^2 = k^2 - 4\pi^2\nu^2 \;; \quad Im(\tilde{\gamma}) \geq 0 \qquad (7)$$

Combining eqs. (2) and (7), it is possible to establish the relationship between the 1D-Fourier transform of the backscattered field and 2D-Fourier transform of the normalized surface current.

$$\hat{K}(\alpha, \beta) = -\frac{2\tilde{\gamma}}{\omega\mu_0} e^{i\tilde{\gamma} y_0} \hat{E}^s(\nu, y_0) \qquad (8)$$

with

$$\alpha(\nu, \theta) = \nu - \frac{k'}{2\pi}\sin\theta \in \mathbf{R}$$

$$\beta(\nu, \theta) = -\frac{1}{2\pi}(\tilde{\gamma} + k\cos\theta) \in \mathbf{R} \; (|\nu| \leq \frac{k'}{2\pi}) \qquad (9)$$

To obtain a usual Fourier transform for K(x,y) , α and β are restricted to their real parts with k'= Re(k), making eq (8) an approximate solution for an embedding dissipative medium.

The 1D-Fourier transform of the backscattered field, $E^s(x,y_0)$, at location y_0 , is defined as

$$\hat{E}^s(\nu, y_0) = \int_{-\infty}^{+\infty} E^s(x, y_0) e^{-2i\pi\nu x} dx \qquad (10)$$

and with the 2D-Fourier transform of the normalized surface current K(x,y)

$$\hat{K}(\alpha, \beta) = \iint_{-\infty}^{+\infty} K(x, y)\ e^{-2i\pi\,(\alpha x + \beta y)}\ dx\ dy \tag{11}$$

The equation (8) is an application of the Fourier Diffraction theorem, which is a generalization of the Radon Projection-slice theorem used in X-ray tomography. This theorem provides information on the 2D-Fourier transform of K in the Fourier space at temporal frequency, ω, on a given semicircle, C(ω), of radius k(ω), whose center is located at -k(ω), as shown in Fig. 2 for normal incidence.

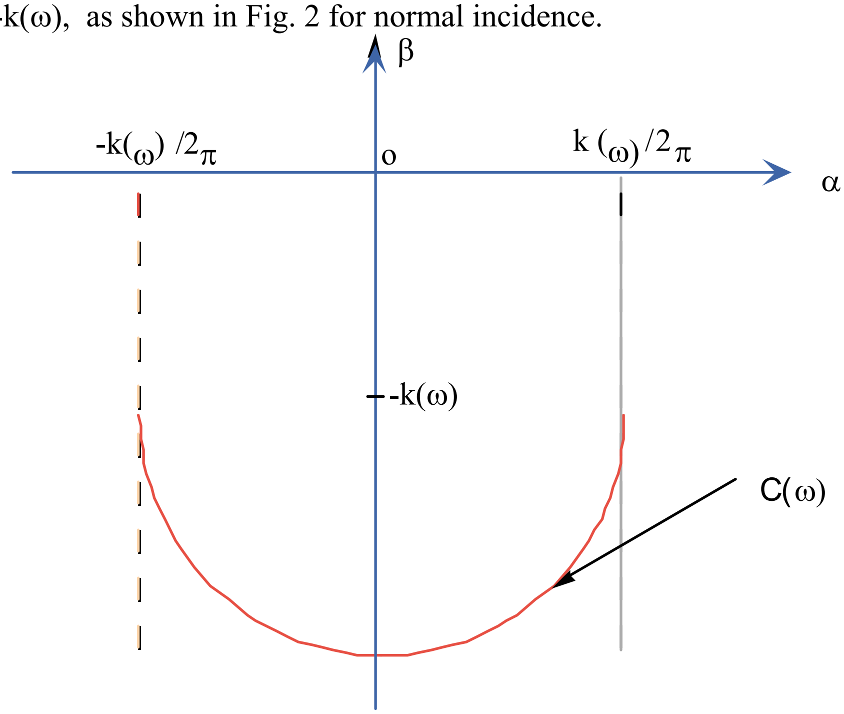

Figure 2 - support of the 2D-Fourier transform of K for normal incidence (θ=0°)

The quality of image reconstruction is improved by processing data obtained at different temporal frequencies in the range $[\omega_{min}, \omega_{max}]$ (Fig.3).

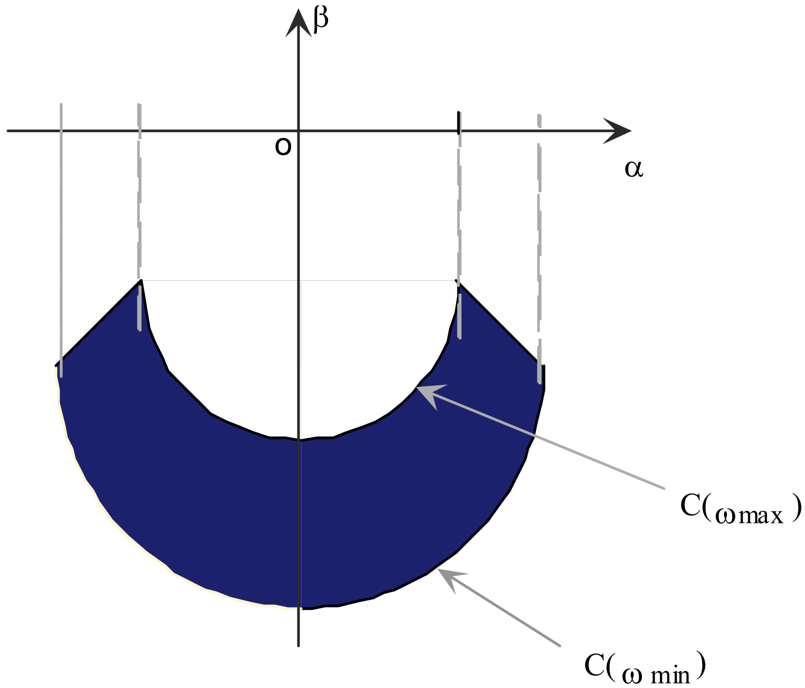

Figure 3 - Variation of the support of the 2D-Fourier transform of K in the frequency range $[\omega_{min}, \omega_{max}]$

Suggestions for further reading:

J.Ch.Bolomey and Ch.Pichot, “Microwave tomography: from theory to practical imaging systems”, *Int. J. Imaging Syst.Tech.*, vol.2, pp. 144-156 (1990).

Bolomey J.Ch. and Pichot Ch. "Some applications of Diffraction Tomogaphy to Electromagnetics- The particular case of Microwaves" in *Inverse Problems in Scattering and Imaging*, edited by M. Bertero and E. R. Pike, Malvern Physics series, Adam Hilger, Bristol, pp. 319-344 (1992).

Bolomey J.Ch., Pichot Ch. and G. Gaboriaud, "Planar microwave imaging for biomedical applications: Critical and prospective analysis of reconstruction algorithms", *Radio Science*, **26** (11), pp. 541-549 (1991).